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THERMAL RADIATION EFFECT ON MHD SLIP FLOW PAST A STRETCHING SHEET WITH VARIABLE VISCOSITY AND HEAT SOURCE/SINK

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ABSTRACT

This paper focuses on a steady two-dimensional slip flow of a viscous incompressible electrically conducting and radiating fluid past a linearly stretching sheet with temperature dependent viscosity is taking into account. The governing boundary layer equations are solved by using Runge-Kutta fourth order technique along with shooting method. The influence of various governing parameters on the fluid velocity, temperature, concentration, skin-friction coefficient, Nusselt number and Sherwood number are computed and discussed in detail.

Keywords: *Radiation, Heat and Mass Transfer, Heat Source/Sink, Magnetic field, Slip Flow, variable viscosity.*

INTRODUCTION:

Boundary layer flow over a moving continuous and linearly stretching surface is a significant type of flow which has considerable practical applications in engineering, electrochemistry and polymer processing, for example, materials manufactured by extrusion processes and heat treated materials travelling between a feed roll and a windup roll or on a conveyor belt possess the characteristics of a moving continuous surface. To be more specific, it may be pointed out that many metallurgical processes involve the cooling of continuous strips or filaments by drawing them through a quiescent fluid and that in the process of drawing, these strips are sometimes stretched. It may be made of drawing, annealing and tinning of copper wires. In all the cases the properties of the final product depend to a great extent on the rate of cooling. By drawing such strips in an electrically conducting fluid subjected to a magnetic field, the rate of cooling can be controlled and a final product of desired characteristics can be achieved. Flow and heat transfer of a viscous fluid past a stretching sheet is a significant problem with industrial heat transfer applications. The steady boundary layer flow of an incompressible viscous fluid due a linearly stretching sheet was investigated by Crane [1]. He obtained an exact similarity solution. The pioneering work of Crane [1] was extended by Pavlov [2]. Pop and Na [3] discussed the unsteady flow due to a stretching sheet. Anderson et al. [4] described the heat transfer in unsteady liquid film over a stretching surface.

Nomenclature

a, b, c	constants	C_∞	ambient concentration
u, v	velocity component	σ_1	Steffan-Boltzmann constant
$x-, y-$	direction component	k_1	mean absorption coefficient
ρ	density	μ^*	constant value of dynamic viscosity
μ	dynamic viscosity of the fluid	υ^*	constant value of kinematic viscosity
T	temperature of the fluid	f	dimensionless stream function
υ	kinematic viscosity	θ	dimensionless temperature
σ	electrical conductivity	ϕ	dimensionless concentration
B_0	uniform transverse magnetic field	A	viscosity parameter
k	thermal conductivity	M	magnetic field parameter
c_p	specific heat constant	N	radiation parameter
q_r	radiative heat flux	$\mathrm{P_r}$	Prandtl number
q	heat source/sink coefficient	Q	non-dimensional heat
D	mass diffusivity		source/sink parameter
L	slip length	S_c	Schmidt number
T_w	temperature at the wall	δ	slip parameter
T_∞	ambient temperature	C_{fx}	skin friction coefficient
C_w	concentration at the wall	N_{ux}	local Nusselt number
Re_x	local Reynolds number	S_{hx}	local Sherwood number

Many recent studies have been focused on the problem of magnetic field effect on laminar mixed convection boundary layer flow over a vertical non-linear stretching sheet [5-7]. Habibi Matin et al. [8] studied the mixed convection MHD flow of nanofluid over a non-linear stretching sheet with effects of viscous dissipation and variable magnetic field. Hamad et al. [9] investigated magnetic field effects on free convection flow of a nanofluid past a vertical semi-infinite flat plate. Kandasamy et al [10] presented the Scaling group transformation for MHD boundary-layer flow of a nanofluid past a vertical stretching surface in the presence of suction/injection.

In all the above mentioned flow problems, the thermophysical properties of fluid were assumed to be constant. However, it is noticed that these properties, especially the fluid viscosity, may change with temperature. In order to appropriately model the flow and heat transfer phenomena, it becomes essential to consider the variation of fluid viscosity due to temperature. Lai and Kulacki [11] considered the effects of variable viscosity on convective heat transfer along a vertical surface in porous medium. Pop et al. [12] discussed the influence of variable viscosity on laminar boundary layer flow and heat transfer due to a continuously moving fiat plate. El-Aziz [13] studied the flow, heat and mass transfer characteristics of a viscous electrically conducting fluid having temperature dependent viscosity and thermal conductivity past a continuously stretching surface, taking into account of the effect of Ohmic heating. Further, some very important investigations regarding the variable viscosity effects on the flow and heat transfer over stretching sheet under different physical conditions were made by Pantokratoras [14], Mukhopadhyay [15, 16]. The non-adherence of the fluid to a solid boundary, known as velocity slip, is a phenomenon that has been observed under certain circumstances. Fluid in micro electro mechanical systems encounters the slip at the boundary. In the previous investigations, it is assumed that the flow field obeys the no-slip condition at the boundary. But, this no-slip boundary condition needs to be replaced by partial slip boundary condition in some practical problems. Beavers and Joseph [17] considered the fluid flow over a permeable wall using the slip boundary condition. The effects of slip at the boundary on the flow of Newtonian fluid over a stretching sheet were studied by Anderson [18] and Wang [19]. Ariel et al. [20] analyzed the flow of a viscoelastic fluid over a stretching sheet with partial slip. Ariel [21] also studied the slip effects on the two dimensional stagnation point flow of an elastoviscous fluid. Bhattacharyya et al. [22] showed the slip effects on the dual solutions of stagnation-point flow and heat transfer towards a shrinking sheet. Recently, Bhattacharyya et al. [23] studied the boundary layer slip flow and heat transfer past a stretching sheet with temperature dependent viscosity. Mukhopadhyay et al. [24] analyzed the effects of temperature dependent viscosity on MHD boundary layer flow and heat transfer over stretching sheet. An extensive literature that deals with flows in the presence of radiation is now available. Cortell [25] has solved a problem on the effect of radiation on Blasius flow by using fourth order Runge-Kutta approach. Later, Sajid and Hayat [26] considered the influence of thermal radiation on the boundary layer flow due to an exponentially stretching sheet by solving the problem analytically via homotopy analysis method (HAM). Bidin and Nazar [27] studied the boundary layer flow over an exponential stretching sheet with thermal radiation, using Keller-box method. Bala Anki Reddy and Bhaskar Reddy [28] analyze the thermal radiation effects on hydro-magnetic flow due to an exponentially stretching sheet. Rafael [29] studied about viscoelastic fluid flow and heat transfer over a stretching sheet under the effects of a non-uniform heat source, viscous dissipation and thermal radiation. Elbashbeshy and Bazid [30] studied the heat transfer over a stretching surface in a porous medium, with internal heat generation and suction or injection. Nagbhooshan [31] analyzes the flow and heat transfer over an exponential stretching sheet under the effects of a temperature gradient dependent heat sink and thermal radiation. Pattnaik et al. [32-38] studied the behaviour of MHD fluid flow and observed some interesting results. However, the interaction of partial slip flow on heat and mass transfer past a stretching sheet immersed in a fluid of variable viscosity, has received little attention. Hence, the present study an attempt is made to analyse a steady magnetohydrodynamic (MHD) slip flow over a stretching sheet in the presence of thermal radiation, heat source/sink and mass transfer. The governing boundary layer equations have been transformed to a two-point boundary value problem in similarity variables and the resultant problem is solved numerically using the fourth order Runge-Kutta method with shooting technique. The effects of various governing parameters on the fluid velocity, temperature, concentration, skin-friction coefficient, Nusselt number and Sherwood number are shown in figures and analysed in detail.

Mathematical analysis:

A steady two-dimensional slip flow of a viscous incompressible electrically conducting and radiating fluid past a linearly stretching sheet with temperature dependent viscosity is considered. The flow is assumed to be in the $x-$direction, which is chosen along the plate in the upward direction and $y-$axis normal to plate. A uniform magnetic field is applied in the direction perpendicular to the plate. The transverse applied magnetic field and magnetic Reynolds number are assumed to be very small, so that the induced magnetic field is negligible.

Under these assumptions, governing equations of the flow field are given by:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{1}{\rho}\frac{\partial \mu}{\partial T}\frac{\partial T}{\partial y}\frac{\partial u}{\partial y}+\upsilon\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_0^2}{\rho}u \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2}-\frac{1}{\rho c_p}\frac{\partial q_r}{\partial y}+\frac{q}{\rho c_p}(T-T_\infty) \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2} \tag{4}$$

$$\begin{aligned} &u=cx+L\frac{\partial u}{\partial y}, v=0, T=T_W, C=C_W \quad \text{at} \quad y=0 \\ &u\to 0, v\to 0, T\to T_\infty, C\to C_\infty \quad \text{as} \quad y\to\infty \end{aligned} \tag{5}$$

Using the Rosseland approximation, the radiative heat flux is given by (Brewster [39])

$$q_r=-\frac{4\sigma_1}{3k_1}\frac{\partial T^4}{\partial y}, \; T^4=4T_\infty^3 T-3T_\infty^4$$

$$\frac{\partial q_r}{\partial y}=-\frac{16T_\infty^3\sigma_1}{3k_1}\frac{\partial^2 T}{\partial y^2}$$

So from equation (3), we have,

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c_p}\left(1+\frac{16T_\infty^3\sigma_1}{3k_1 k}\right)\frac{\partial^2 T}{\partial y^2}+\frac{q}{\rho c_p}(T-T_\infty) \tag{6}$$

The continuity equation (1) is satisfied by

$$u=\frac{\partial \psi}{\partial y}, v=-\frac{\partial \psi}{\partial x} \tag{7}$$

where $\psi(x,y)$ is the stream function.

The temperature dependent viscosity of the fluid is of the form

$$\mu = a\mu^* + b(T_w - T) \quad a,b(>0) \tag{8}$$

Here, $\mu = e^{-a^*T}$ and $\upsilon = a\upsilon^* + b(T_w - T)$ where $\upsilon^* = \dfrac{\mu}{\rho}$ when second and higher order terms are neglected from the expansion.

With the help of stream function and the following similarity transformation:

$$\eta = y\sqrt{\frac{c}{\upsilon^*}}, \psi = \sqrt{c\upsilon^*}\, xf(\eta), \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}$$

Eqs. (2), (6) and (4) can be written as,

$$(a + A - A\theta) f''' + ff'' - f'^2 - A\theta' f'' - Mf' = 0 \tag{9}$$

$$(1+N)\theta'' + P_r f\theta' + P_r Q\theta = 0 \tag{10}$$

$$\phi'' + S_c f\phi' = 0 \tag{11}$$

where $A = b(T_w - T_\infty), M = \dfrac{\sigma B_0^2}{\rho c}, N = \dfrac{16\sigma_1 T_\infty^3}{3kk_1}, \mathrm{P_r} = \dfrac{\mu^* c_p}{k}, Q = \dfrac{q}{c\rho c_p}, S_c = \dfrac{\upsilon}{D}$

So the boundary conditions are reduced as:

$$\begin{aligned} &f = 0, f' = 1 + \delta f'', \theta = 1, \phi = 1 \quad \text{at} \quad \eta = 0 \\ &f' \to 0,\ \theta \to 0,\ \phi \to 0 \qquad \text{as} \quad \eta \to \infty \end{aligned} \tag{12}$$

where $\delta = L\sqrt{c\upsilon^*}$.

Physical quantities:

Skin friction coefficient, local Nusselt number and local Sherwood number respectively are defined as

$$C_{fx}\sqrt{\frac{\mathrm{Re}_x}{2}} = f''(0), N_{ux} / \sqrt{\frac{\mathrm{Re}_x}{2}} = -\theta'(0), S_{hx} / \sqrt{\frac{\mathrm{Re}_x}{2}} = -\phi'(0) \tag{13}$$

where $\mathrm{Re}_x = \dfrac{xU_w(x)}{v}$.

Thus the values whivh are proportional to the Skin-friction coefficient, Nusselt number and the Sherwood number are $f''(0), -\theta'(0)$ and $-\phi'(0)$ respectively.

Method of solution

The set of coupled non-linear governing boundary layer equations (2) - (4) together with the boundary conditions (5) are solved numerically by using Runge-Kutta fourth order technique along with shooting method. First of all, the higher order non-linear partial differential equations (2) - (4) are converted into simultaneous linear coupled differential equations of first order and they are further transformed into an initial value problem by applying the shooting technique (Jain et al. [40]). The resultant initial value problem is solved by employing Runge-Kutta fourth order technique. By the help of graph, the skin-friction coefficient, the Nusselt number and the Sherwood number, which are respectively proportional to $f''(0), -\theta'(0)$ and $-\phi'(0)$ are also sorted out.

Results and discussion

In order to get a clear insight of the physical problem, the velocity, temperature and concentration profiles have been discussed by assigning numerical values to the governing parameters encountered in the problem. The effects of various parameters on the velocity are depicted in Figs. (1) & (2). The effects of various parameters on the temperature are depicted in Figs. (3) & (4). The effects of various parameters on the concentration profile are depicted in Fig. (5). In the present computation the value of the pertinent parameters are considered as $A = M = N = Q = \delta = \mathrm{P_r} = S_c = 1$ unless otherwise stated.

Fig.1 (a-c) shows the variation of velocity profile for different values of A, M and N. It is observed from Fig. 1(a) that the velocity increases with increasing values of viscosity parameter (A) and hence the momentum boundary layer thickness increases. Fig. 1(b) shows the variation of velocity profiles for different values of magnetic parameter (M). It is seen that, as expected, the velocity decreases with an increase of magnetic parameter. The magnetic parameter is found to retard the velocity at all points of the flow field. It is because that the application of transverse magnetic field will result in a resistive type force (Lorentz force) similar to drag force which tends to resist the fluid flow and thus reducing its velocity. Also, the boundary layer thickness decreases with an increase in the magnetic parameter. The effect of radiation parameter (N) on the velocity is illustrated in Fig. 1(c). It is noticed that the velocity profile increases with increasing values of the radiation parameter. Fig. 2(a-c) shows the variation of velocity profile for different values of Q, δ and P_r. Fig. 2(a) illustrates the effect of heat source/sink parameter (Q) on the velocity. It is noticed that as the heat source/sink parameter increases, the velocity profile increases. Fig. 2(b) shows the variation of velocity profile for different values of the slip parameter (δ) on the velocity field. The flow is decelerated due to the enhancement in the slip parameter. Fig. 2(c) is the evidence of enhanced velocity boundary layer thickness due to the increased values of Prandtl number (P_r).Fig. 3(a-c) shows the variation of temperature profile for different values of A, M and δ. Fig. 3(a) shows the variation of temperature profile for different values of viscosity parameter (A). It is observed that the temperature decreases with increasing values of (A) and the thermal boundary layer thickness decreases with (A).The effect of the magnetic parameter on the temperature is illustrated in Fig. 3(b). It is observed that as the magnetic parameter (M) increases, the temperature increases. The effect of the slip parameter (δ) on the temperature is depicted in Fig. 3(c). It is seen that as the slip parameter increases, the temperature increases. Fig. 4(a-c) shows the variation of temperature profile for different values of P_r, N and Q. Fig. 4(a) depicts the variation of the thermal boundary-layer with the Prandtl number (P_r).It is noticed that the thermal boundary layer thickness decreases with an increase in the Prandtl number. Fig. 4(b) shows the variation of the thermal boundary-layer with the radiation parameter (N). It is observed that the thermal boundary layer thickness increases with an increase in the radiation parameter. Fig. 4(c) shows the accelerated direction of temperature profile for increasing values of heat source/sink parameter. Fig. 5(a-c) shows the variation of concentration profile for different values of A, M, δ and S_c .Fig. 5(a) shows that the concentration profile decreases with increasing values of viscosity parameter. The effect of magnetic parameter on the concentration field is illustrated in Fig. 5(b). As the magnetic parameter increases the concentration is found to be increasing. In Fig. 5(c) it is clear that for increasing values of slip parameter, the concentration profile gets accelerated but reverse effect has been observed for increasing values of Schmidt number in Fig. 5(d).Skin friction coefficient is increased for increasing values of A, Q and N but reverse trend is noticed for M in Fig. 6(a-d). Nusselt number, on the other hand, increases for increased A but for increasing values of M, Q and N, it decreases as shown in Fig.7 (a-d). Sherwood number is found to decrease with increasing values of A, M, Q and S_c which is noticed in Fig. 8(a-d). The present results are compared with those of Anderson [18] and Bhattacharya et al. [22] and found that there is a perfect agreement.

Conclusion:

In the present study, the steady boundary layer slip flow and heat transfer past a stretching sheet with temperature dependent viscosity is considered in the presence of thermal radiation, heat source/sink and mass transfer. The governing equations are approximated to a system of non-linear ordinary differential equations by similarity transformations. Numerical calculations are carried out for various values of the dimensionless parameters of the problem. The present solutions are validated by comparing with the existing solutions. Our results show a good agreement with the existing work in the literature.

The results are summarized as follows:

- The viscosity parameter enhances the velocity and reduces and the temperature concentration.
- Magnetic field elevates the temperature and concentration and reduces the velocity.
- The radiation enhances the velocity and temperature.
- The heat source/sink enhances the velocity and temperature.
- The radiation parameter elevates the skin friction and reduces the heat transfer.

Fig.1 Variation of Velocity profile f '(η)

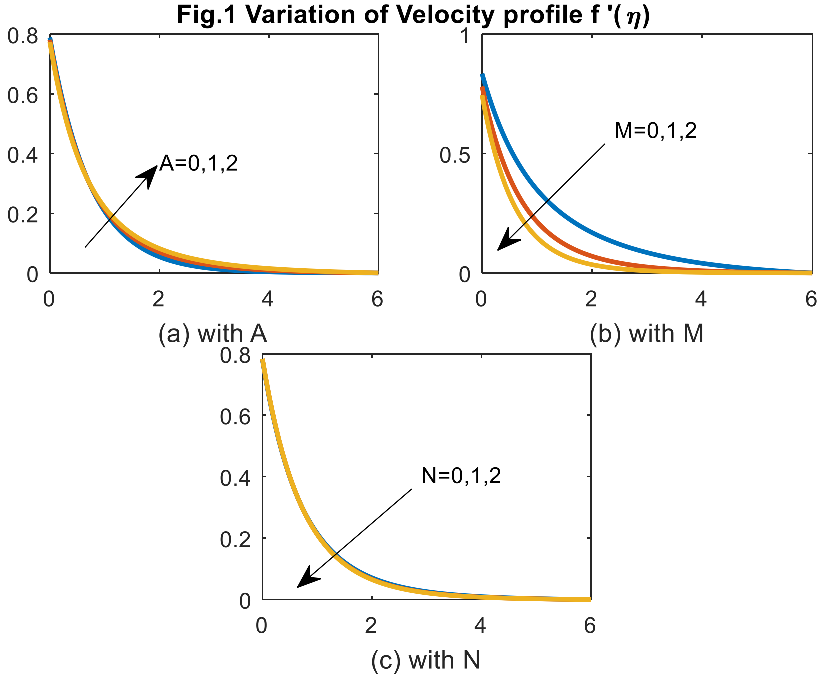


Fig.2 Variation of Velocity profile f '(η)

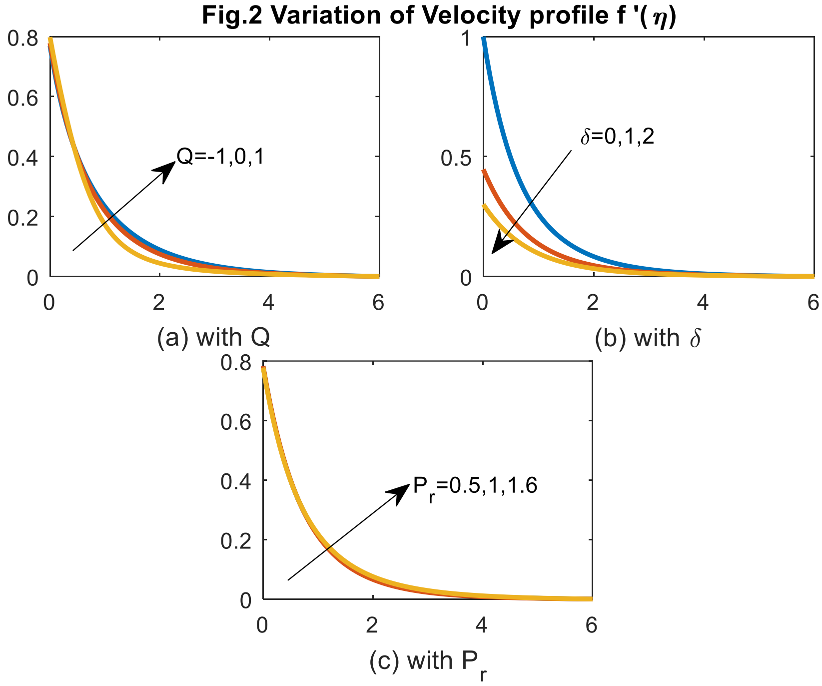


Fig.3 Variation of Temperature profile $\theta(\eta)$

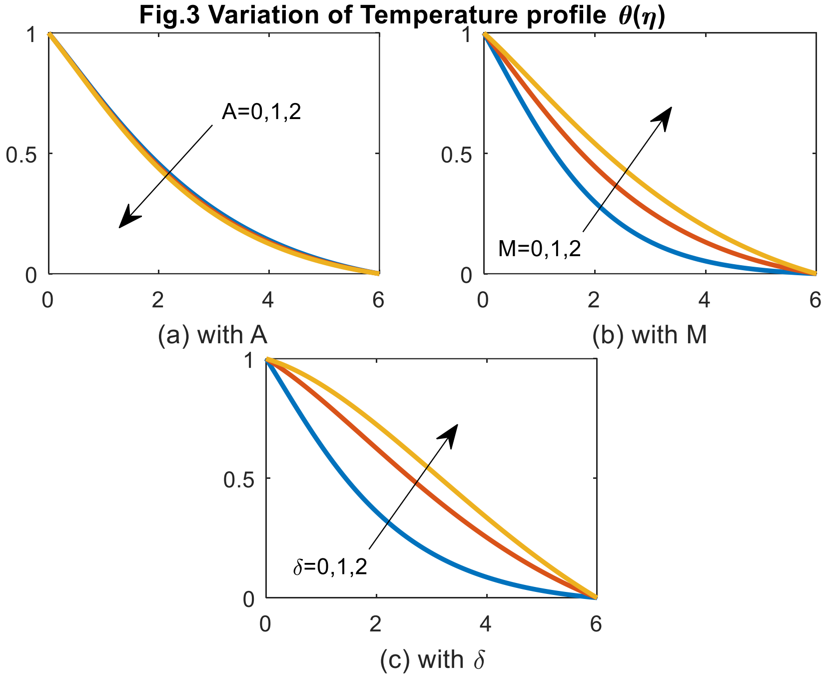


Fig.4 Variation of Temperature profile $\theta(\eta)$

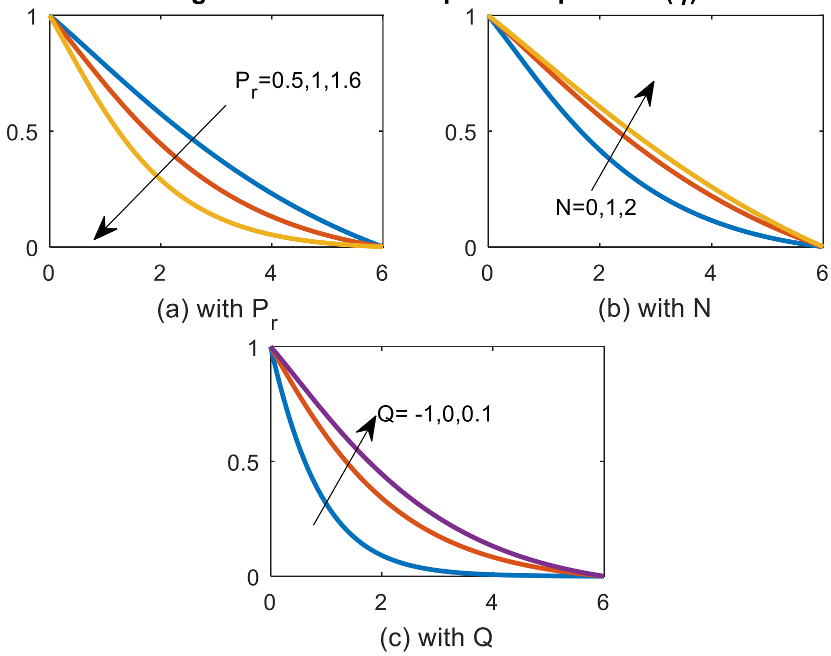


Fig.5 Variation of Concentration profile $\phi(\eta)$

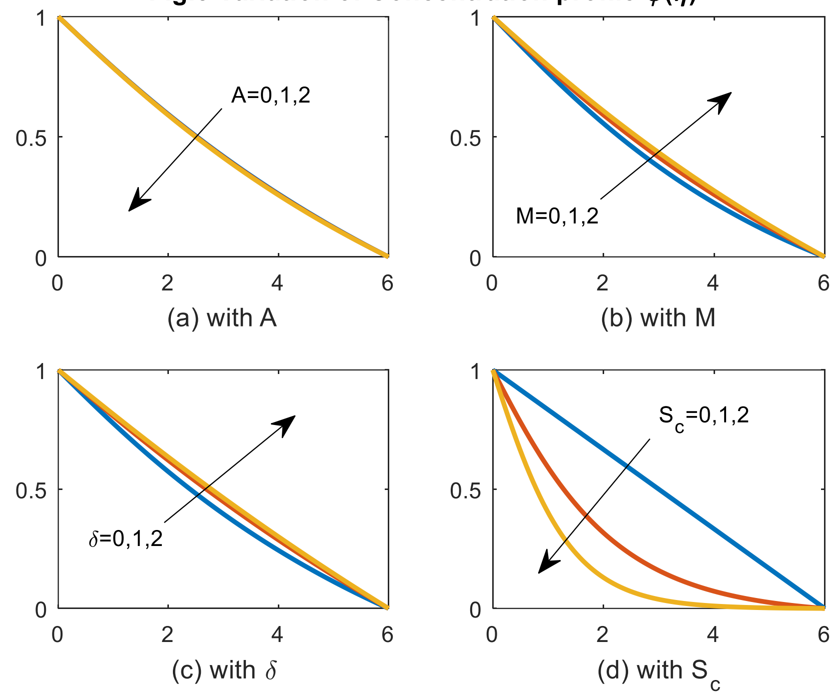


Fig.6 Variation of Skin friction coefficient C_f

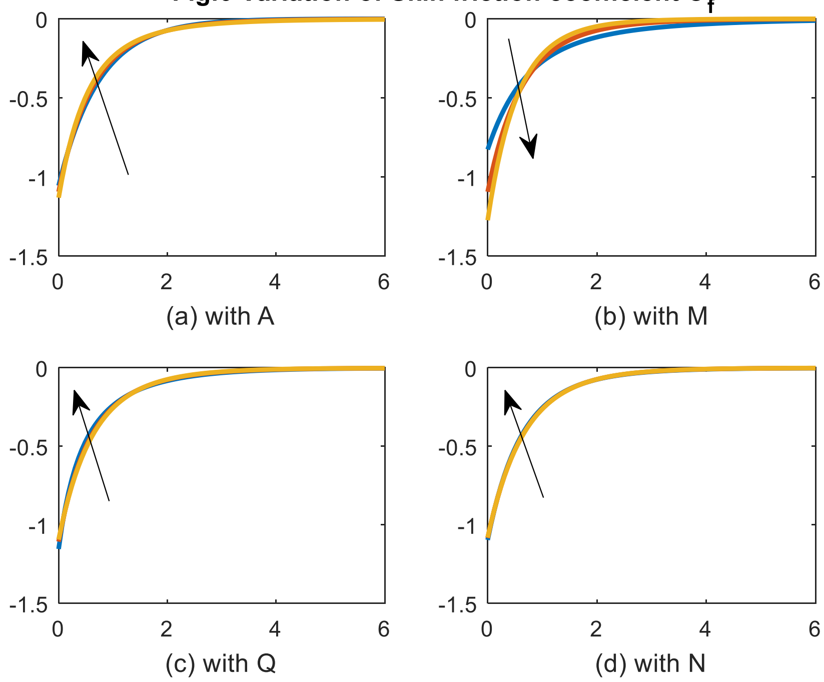


Fig.7 Variation of Nusselt number Nu_x

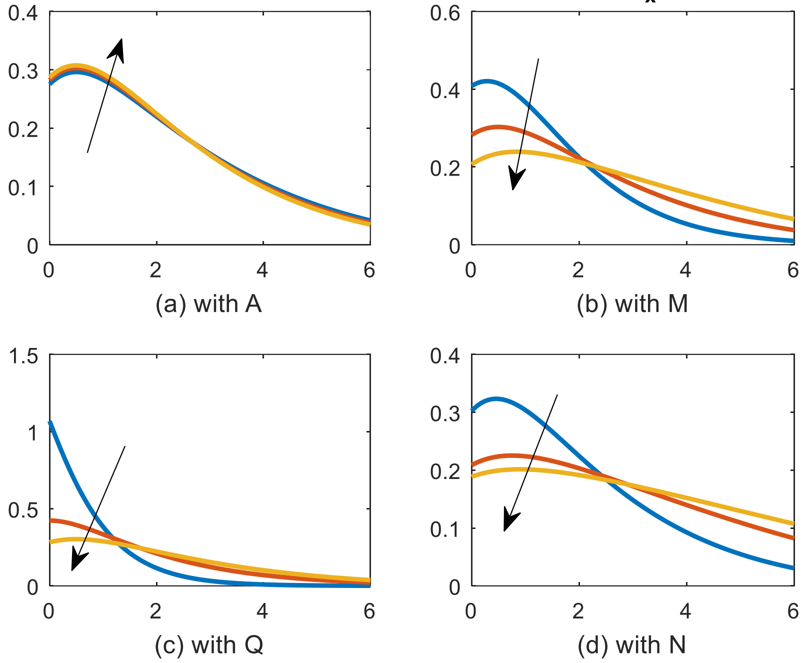
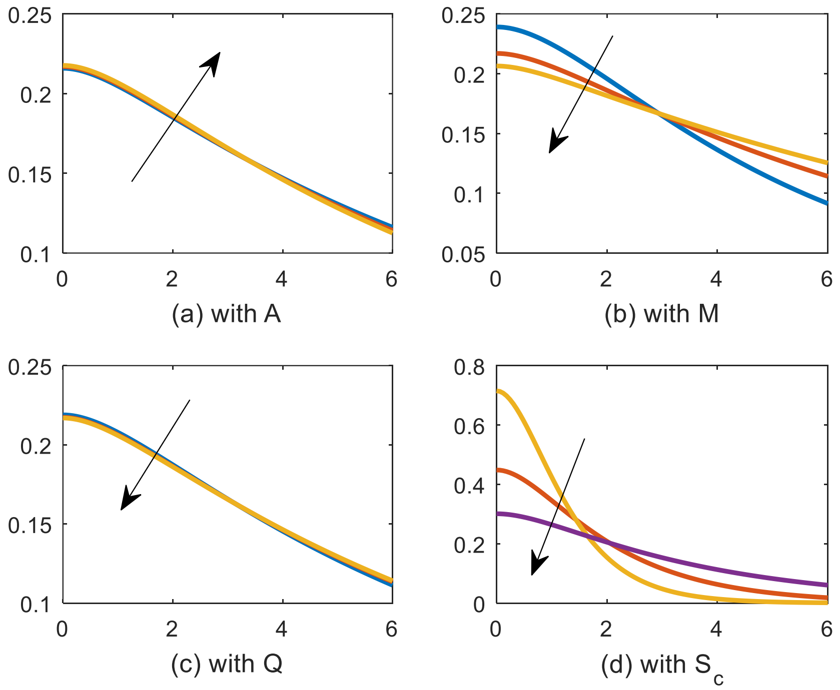


Fig.8 Variation of Sherwood number Sh_x



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