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CONSTRUCTION OF DIFFERENT TYPES OF CONTROL CHARTS FOR LENGTH OF STAY OF PATIENTS IN HEALTHCARE ORGANISATIONS

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ABSTRACT

Traditional Shewhart control charts assume normality and does not consider the skewness of statistics under study. This paper proposes control charts based on skewness and kurtosis of patient's length of stay. The asymmetric control limits are based on the degree of skewness and kurtosis estimated from the sample . These charts are simply adjustments of the conventional Shewhart control charts. The new charts are compared with the Shewhart chart and evaluated based on performance measure false alarm rate.

Key words *: Skewness ,Skewness correction, Control charts ,False alarm rate, Kurtosis correction,Skewed distribution, Length of stay.*

INTRODUCTION

Statistical process control (SPC) is a branch of statistics that uses different methods and represents data graphically, often yielding insights into the data more quickly and in a way more understandable to lay decision makers. SPC is an effective and powerful graphical tool used to separate the variation due to chance and assignable causes. Its primary tool—the control chart—provide researchers and practitioners with a method of better understanding the data.

Healthcare is an extremely important sector in service industry, as it is related to human lives and there will always be a scope for all the healthcare systems (hospitals) to improve and provide good quality service.

1.1 Why length of stay?

In hospital's quality management there are many variables like surgery time, elapse time, length of stay, type of discharge etc. Out of which the variable length of stay (LOS) of patients admitted is selected here.

For any hospital, length of stay is one of the main measures of performance. The average length of stay in hospitals (ALOS) is often used as an indicator of efficiency.

Patient's length of stay(LOS) is one of the biggest issues hospitals facing today. The longer a patient stays in the hospital, the greater is the risk they will develop a healthcare acquired infection (HAI) that they can become vulnerable to. In addition, hospitals face lower patient capacities and increased costs.

Hospitals benefit from a shorter LOS. They do not have to cover the expense of treating an HAI. All other things being equal, a shorter stay will reduce the cost per discharge. Also hospitals free up beds for new patients allowing them to treat more patients. While it sounds relatively simple to reduce length of stay LOS, in reality, this issue is fraught with obstacles. Here's a look at how hospitals can work towards decreasing LOS, saving money in the process by using available technology to become more efficient.

Considering these factors, in this paper control charts for the r.v. LOS are developed and an attempt to increase hospital profitability because of good quality service is made to improve customer satisfaction.

Traditional Shewhart control chart assumes normality and does not consider the skewness of the statistic under study. More often the skewness is too large to ignore. In such situations traditional control charts are improper to give satisfactory performance and may give erroneous conclusions. The use of traditional Shewhart control chart in skewed distribution causes an increase in type I risk with an increase in skewness. For this reason some methods which use asymmetric control limits were proposed as an alternative to the classical method.

Therefore, this paper proposes control charts based on skewness and kurtosis of patients length of stay in hospital. The asymmetric control limits are based on the degree of skewness estimated from the subgroups. These charts are simply adjustments of the conventional Shewhart control charts. The new charts are compared with the Shewhart charts and evaluated on the basis of performance measure false alarm rate.

The first chart is constructed using the skewness correction (SC) method proposed by Chan and Cui (2003). The second chart is constructed using kurtosis correction (KC) method proposed by Tadikamalla and Popescu(2007).

1.2 Distribution of length of stay (LOS)

To construct control charts for LOS,the distribution for LOS is required. LOS can be considered as a discrete or continuous r.v, depending upon how hospitals record the length of stay or how they charge for different length of stays. The empirical distribution of LOS is often positively skewed.

Let us consider the discrete case where LOS has Poison distribution with parameter λ. i.e.
$LOS \sim P(\lambda)$; LOS=0,1,2,3…….. ; $\lambda > 0$
$E[LOS] = \lambda$
$V[LOS] = \lambda$

Here LOS = 0, indicates that patient stays for less than six hours in hospitals. For instance patient admitted given a saline, prescribed medicines and discharged with some instruction etc. The criteria for zero length of stay differs from hospital to hospital. Around a third of all emergency admissions are now zero day stays. This means that the patient does not need to stay overnight.

2. Shewhart Control Chart

The control limits of conventional Shewhart method are given by

$UCL = E(LOS) + 3\sqrt{V(LOS)}$
$CL = E(LOS)$
$LCL = E(LOS) - 3\sqrt{V(LOS)}$

Assuming Poisson distribution of r.v. LOS the control limits by Shewhart control chart are given as follows.

$$UCL = \lambda + 3\sqrt{\lambda} \qquad (1)$$

$CL = \lambda$
$LCL = \lambda - 3\sqrt{\lambda}$

2.1. False alarm rate (FAR)

Let α_u denote the type I probability generated in the upper tail which is given by
$\alpha_u = P[LOS > UCL] = 1 - P[LOS <= UCL]$

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Let α_l denote the type I probability generated in the lower tail which is given by
$\alpha_l = P[LOS < LCL]$

Therefore false alarm rate $= \alpha = \alpha_u + \alpha_l$

Since we are finding control limits for LOS and we are interested in reducing it, there is no meaning in defining lower control limit for this chart. However upper control limit obviously makes sense since it will enable monitoring improbable high values of LOS. Therefore in this chart and in the the Skewness Corrected (SC) and kurtosis Corrected(KC) control charts only UCL & α_u are computed.

Hence

$$\text{False alarm rate } = \alpha = \alpha_u \qquad (2)$$

2.2. ARL for Shewhart control chart

Let the Average run length be denoted by ARL_{LOS}.

$$ARL_{LOS} = \frac{1}{\alpha_u + \alpha_l} = \frac{1}{\alpha_u} \qquad (3)$$

2.3. Numerical analysis of Shewhart control chart

In the Following table, UCL gives the upper control limits for length of stay(LOS) for different values of λ using Shewhart control chart with L=3 and is calculated using (1). alphaU is the FAR for Shewhart control chart calculated using (2) and ARLfor this is calculated using (3)

Table 1 : UCL, FAR and ARL of Shiwhart control chart with L=3 for different values of λ

λ	UCL (expressed in days)	alphaU	ARL
1	4	0.0037	273
2	6	0.0045	221
3	8	0.0038	263
4	10	0.0028	352
5	12	0.0055	183
6	13	0.0036	276
7	15	0.0057	175
8	16	0.0037	269

9	18	0.0024	412
10	19	0.0035	289
11	21	0.0047	214
12	22	0.003	328
13	24	0.004	252
14	25	0.0026	383
15	27	0.0033	302
16	28	0.0022	457
17	29	0.0027	367
18	31	0.0033	300
19	32	0.0022	449
20	33	0.0027	372

2.4. CONCLUSION

It can be observed from the above table that as Length of Stay (λ) increases UCL increases but no trend is exhibited by α_u and ARL. The values of α_u and ARL keeps fluctuating.

3. Control Chart For LOS Using Skewness Correction(SC) Method

This method is used for constructing control charts, considering the degree of skewness of the process distribution, with no assumption on the distribution. This method, called skewness correction (SC) method. It corrects the conventional Shewhart chart according to the skewness of the distribution. It provides asymmetric control limits using ±3 standard deviation plus the known function of degree of skewness, $\frac{\frac{4}{3}*skew(LOS)}{1+0.2*skew^2(LOS)}$ "Chan and Cui (2003)". This chart reduces to the Shewhart chart for symmetric distributions i.e. when coefficient of skewness $\beta_1 = 0$.

The control limits using SC method for patients LOS are given by

UCL = E(LOS) + $(3+C_4^*)\sqrt{V(LOS)}$
CL = E(LOS)
LCL = E(LOS)+(- 3+ $C_4^*)\sqrt{V(LOS)}$
Here C_4^* is a control chart constant for skewness corrected method and is given by

$$C_4^* = \frac{\frac{4}{3}*skew(LOS)}{1+0.2*skew^2(LOS)} \quad (4)$$

It is noted that these control limits differ from those of the standardized Shewhart control chart by the amount of skewness correction, $\frac{\frac{4}{3}*skew(LOS)}{1+0.2*skew^2(LOS)}$

Assuming Poisson distribution of r.v. LOS,

Skewness (LOS) = $\beta_1 = \frac{\mu_3^2}{\mu_2^3} = \frac{\lambda^2}{\lambda^3} = \frac{1}{\lambda}$

and the control limits for skewness correction method are given as follows.

$$UCL = \lambda + (3+C_4^*)\sqrt{\lambda} \quad (5)$$

CL = λ

LCL = $\lambda + (-3 + C_4^*)\sqrt{\lambda}$

3.1. False alarm rate (FAR)

Let α_u be the type I probability generated in the upper tail

α_u = P[LOS > UCL] = 1-P[LOS<= UCL]

Let α_l be the type I probability generated in the lower tail

α_l = P[LOS < LCL]

FAR = $\alpha = \alpha_u + \alpha_l = \alpha_u$

3.2. ARL for control chart using skewness correction method

Average run length for LOS is denoted by ARL_{LOS} and is given by

$$ARL_{LOS} = \frac{1}{\alpha_u + \alpha_l} = \frac{1}{\alpha_u}$$

3.3. Numerical analysis of control chart for LOS using skewness correction(SC) method

In the Following table, UCLsc gives the upper control limit of control chart for Length Of Stay(LOS) with L=3 for different values of λ using skewness correction method(SC) and is calculated using (5). alphaUsc is the FAR for this control chart and ARLsc is the average run length for this control chart.

Table 2. UCL, FAR and ARL of the control chart for LOS with L=3 and for different values of λ using Skewness Correction method

λ	UCL_{sc} (Expressed in DAYS)	$alphaU_{sc}$	ARL_{sc}

1	5	6e-04	1683
2	7	0.0011	912
3	9	0.0038	263
4	11	0.0028	352
5	12	0.002	495
6	14	0.0036	276
7	15	0.0024	416
8	17	0.0037	269
9	18	0.0024	412
10	20	0.0035	289
11	21	0.0023	444
12	23	0.003	328
13	24	0.002	501
14	26	0.0026	383
15	27	0.0033	302
16	28	0.0022	457
17	30	0.0027	367
18	31	0.0018	551
19	32	0.0022	449
20	34	0.0027	372

3.4. CONCLUSION

It can be observed from the above table that as Length of Stay (λ) increases UCL increases but no trend is exhibited by α_u and ARL. The values of α_u and ARL keeps fluctuating.

4. Control Chart For LOS Using Kurtosis Correction (KC) Method

Kurtosis correction method was developed by Tadikamalla and Popescu. It is used to construct control chart when the process distribution is symmetrical, but is leptokurtic. This method has the advantage of providing better control limits and are easier to use and make no assumption on the functional form of underlying distribution. This method shifts the control limits to both sides by the same amount $\frac{kurt(LOS)}{1+0.33*kurt(LOS)}$ "Tadikamalla and Popescu(2007)" which is function of kurtosis. This control chart reduces to the Shewhart chart when kurtosis of underlying distribution is zero *i.e.* when coefficient of kurtosis $\beta_2 = 0$.

The control limits for kurtosis corrected (KC) method are given as follows.

$$UCL = E(LOS) + (3+C_5)\sqrt{V(LOS)} \quad (6)$$

CL = E(LOS)

LCL = E(LOS) – (3+ C_5)$\sqrt{V(LOS)}$

Here C_5 is a control chart constant for kurtosis corrected chart and is given by

$C_5 = \frac{kurt(LOS)}{1+0.33*kurt(LOS)}$ (7)

Where kurt(LOS) is kurtosis of Length of stay.

As LOS ~ P(λ)

Therefore kurt(LOS)= Kurtosis coefficient $\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{3\lambda^2+\lambda}{\lambda^2} = 3 + \frac{1}{\lambda}$

4.1. False alarm rate

Let α_u is the type I probability generated in the upper tail

α_u = P[LOS > UCL] = 1-P[LOS<= UCL]

Let α_l = type I probability generated in the lower tail

α_l =P[LOS < LCL]

FAR = $\alpha = \alpha_u + \alpha_l = \alpha_u$

4.2. ARL for control chart using kurtosis correction (KC) method

Let the Average run length be denoted by ARL_{LOS}.

$$ARL_{LOS} = \frac{1}{\alpha_u + \alpha_l} = \frac{1}{\alpha_u}$$

4.3. Numerical analysis of control chart for LOS using Kurtosis Correction(KC) method:

In the Following table, UCLkc gives the upper control limit of control chart for Length Of Stay(LOS) with L=3 for different values of λ using kurtosis correction(KC) method and is calculated using (6). alphaUkc is the FAR for this control chart and ARLkc is the average run length for this chart.

Table 3. UCL, FAR and ARL of the control chart for LOS with L=3 and for different values of λ using Kurtosis Correction method

λ	UCL_{kc} (Expressed in DAYS)	$alphaU_{kc}$	ARL_{kc}
1	6	6e-04	1683
2	9	0.0011	912

3	11	0.0038	263
4	13	0.0028	352
5	15	0.002	495
6	17	0.0036	276
7	19	0.0024	416
8	21	0.0037	269
9	23	0.0024	412
10	24	0.0035	289
11	26	0.0023	444
12	28	0.003	328
13	29	0.002	501
14	31	0.0026	383
15	33	0.0033	302
16	34	0.0022	457
17	36	0.0027	367
18	37	0.0018	551
19	39	0.0022	449
20	40	0.0027	372

4.4. Conclusion

It can be observed from the above table that as Length of Stay (λ) increases UCL increases but no trend is exhibited by α_u and ARL. The values of α_u and ARL keeps fluctuating.

5. Comparison Of Control Chart For LOS Using Shewhart Method ,Skewness Corrected Method And Kurtosis Corrected Method

Following table shows ARL of control charts calculated using Shewhart method ,Skewness corrected method and Kurtosis corrected method

Table 4 : ARLs for different values of λ obtained using Shewhart, Skewness Correction And Kurtosis Correction Method

λ	ARL	ARL_{sc}	ARL_{kc}
1	273	1683	1683
2	221	912	912
3	263	263	263
4	352	352	352
5	183	495	495
6	276	276	276

7	175	416	416
8	269	269	269
9	412	412	412
10	289	289	289
11	214	444	444
12	328	328	328
13	252	501	501
14	383	383	383
15	302	302	302
16	457	457	457
17	367	367	367
18	300	551	551
19	449	449	449
20	372	372	372

5.1 CONCLUSIONS :

From table 4 , it may be observed that for different values of λ,

1. The ARL of control chart based on skewness correction method is greater than or equal to the ARL of Shewhart control chart. Therefore it may be concluded that , control chatr based on skewness correction method is better than Shewhart control chart.
2. The ARL of control chart based on kurtosis correction method is greater than or equal to the ARL of Shewhart control chart. Therefore it may be concluded that the control chart based on kurtosis correction method is better than Shewhart control chart.
3. The average run length(ARL) of control charts based on skewness correction method and kurtosis correction method is same.

6. Interpretation Of Out Of Control Signal In Control Chart Related To LOS In Hospital

Length of stay is commonly used as a quality metric in healthcare. It can be linked to additional quality metrics such as patient satisfaction with health professionals, reduction in hospital readmissions, and even mortality. Discharge planning processes can be effective in reducing a patients length of stay in hospital. For example, for older people admitted with a medical condition, discharge planning has been shown to improve satisfaction.

It is clear that a lower LOS is always better for patients as well as healthcare organisations. Patients to reduce the risk of getting hospital acquired infections and also the waiting time is decreased for other patients in need of a room to receive care, there by increasing the customer satisfaction and in turn hospitals can free up more beds allowing them to treat more patients thereby increasing the profitability and also can provide services at a comparatively lower cost.

So whenever LOS plots outside the control limits as indicated by control charts, then it indicates that, the hospitals should try to minimize the LOS by focusing on the factors affecting it, like delay in laboratory exams, hospital hygiene, discharge process, insurance claims, optimum utilization of equipments *etc.*

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